As promised, this month's paper by Norman Neidell is the first in a series of four where the author gradually leads us towards a new approach for seismic acquisition and processing. His novel views on spatial sampling are quite challenging and have the merit of making us rethink simple concepts that we have perhaps taken too much for granted. For example, the Nyquist theorem is not as restrictive as many people believe, which could possibly lead to more cost-effective 3-D surveys.

Norm generated a lot of interest and controversy when he presented some of these ideas at the workshop on seismic acquisition just prior to SEG's 1996 Annual Meeting. These papers are likely to generate the same amount of debate. This column welcomes controversy (as long as it has something to do with seismic acquisition or processing) and will host any contribution that challenges or supports new theories.

Since the four papers represent a whole, please do not respond too quickly to Norm; wait for the final paper to be published before starting a discussion. As editor of this column, I have had the privilege of reading the first three papers and I hope you will find them as challenging as I did. In fact, I am now eagerly looking forward to No. 4.

– Guillaume Cambois

Perceptions in seismic imaging Part 1: Kirchhoff migration operators in space and offset time, an appreciation

Norman S. Neidell, N.S. Neidell & Associates, Houston, Texas

The wave equation as it applies to seismic analysis has at least one more variable than we can comfortably visualize (time, or perhaps one of the spatial coordinates depending on what we choose to draw to represent the physics). For that reason, many technical workers who rely on diagrams or other graphic displays to help develop their insights and intuitions may find themselves misdirected occasionally even when considering ideas which in themselves appear simple. This note calls attention to one frequently occurring perception with the intent both of showing its validity while at the same time introducing more cautionary elements into all of our considerations. In particular, we will look at one of the more common so-called Kirchhoff methods of seismic imaging to help attain a more comprehensive understanding of what is involved.

We shall examine the notion that recorded seismic amplitudes may be distributed over elliptical trajectories or ellipsoidal surfaces of equal two-way arrival time for a separated source and receiver as a migration operation.

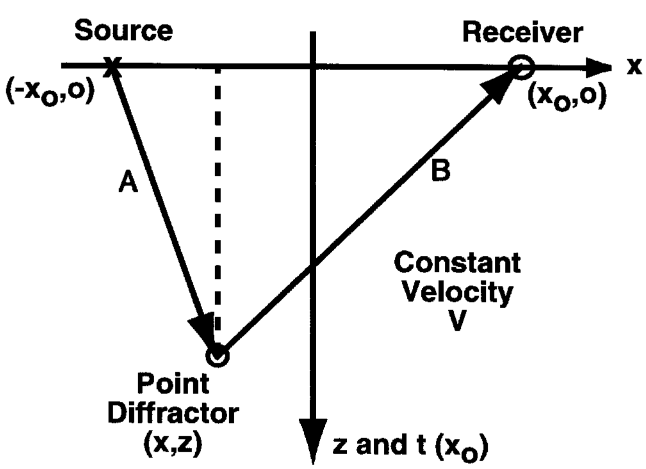

Figure 1. Point diffractor and raypaths for calculating traveltime.

Circumstances when this notion applies will be identified. Many should find the conclusions both surprising and instructive.

Tutorial treatments of very basic seismic operations are not as often seen these days as they should be. Let us then, for the purposes of illustration, show that for a two-dimensional case (coordinates x and z), the locus of a traveltime trajectory in a constant-velocity half-space is indeed elliptical for a separated source and receiver at the surface of the half-space. For this exercise, let us note Figure 1 and the quantities which it implicitly defines.

A presumed point reflector on the defined trajectory has coordinates (x,z). Following more-or-less standard practice, the origin (0,0) is midway between source and receiver which are, therefore, separated by the distance $2x_0$. For the assumed constant velocity v, the constant travel time t_c in this case is simply (A+B)/v where A and B are components of the reflection travel path as indicated in Figure 1. For traveltime t_c, we have

$$t_c = \frac{A+B}{V} = \frac{1}{V}\sqrt{(x+x_0)^2+z^2} + \frac{1}{V}\sqrt{(x-x_0)^2+z^2}$$

which can be written as

$$\left(\frac{V^2t_c^2}{2} - x^2 - x_0^2 - z^2\right)^2 = x^4 + x_0^4 + z^4 - 2x_0^2x^2 + 2z^2x^2 + 2z^2x_0^2$$

and then simplified to

$$4x^2x_0^2 - V^2t_cx^2 - V^2t_c^2z^2 = -\frac{V^4t_c^4}{4} + V^2t_c^2x_0^2$$

which is really the standard form of an ellipse

$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$$

if a and b are defined as

$$a = \frac{Vt_c}{2} \text{ and } b = \frac{1}{2}\sqrt{V^2t^2_c - 4x_0{}^2}$$

Of course, a and b are the semi-major and semi-minor axes of the ellipse. Note that for $x_0 = 0$ (a coincident source and receiver), a = b and our ellipse would become a circle (as we know it should).

It is obvious that an ellipse is generated when we are working with a coordinate system involving *x* and *z*. Is this always the case? Or will we get some other curve in a coordinate system which paired *x* with a time variable? This is not quite as straightforward a question as it may appear to be. After all, the time which might be chosen should be one which is measurable, and such a coordinate would necessarily have values depending also on the separation of the source a receiver. Also, since there would be no *x* value uniquely relating to such a time measurement (since two surface positions, source and receiver, are involved), one must be chosen. Our usual convention is to pick a value midway between the source and receiver.

If we redraw Figure 1 with a time axis rather than a *z* axis, t_c (the previously selected constant traveltime) would not be related to a *z* value by only a scaling of the velocity. It is an easy exercise to show a *z* value (which we shall call z_0) is related to t_c by the equation

$$z_0 = \frac{1}{2}\sqrt{V^2t^2_c - 4x_0{}^2}$$

which is the quantity previously called b.

As it happens, the choice of plotting convention for the recorded arrival times as defined is a happy one. From the definition implicit in the above equation, it follows that the variables *z* and *t* (which has to be a function of source-receiver separation, $2x_0$) can be related by the ratios

$$\frac{z}{b} = \frac{t}{t_c}$$

Solving for *z* and then substituting for *z* in the previous work will show that in *x-t* space, Kirchhoff migration is also accomplished along elliptical trajectories.

Note that the t variable in Figure 1, as it might have been redrawn, is indicated to be a function of x_0, a parameter relating to the source-receiver offset or separation. It is actually a family of *x-t* spaces. Amplitude values at each $(x, t(x_0))$, space can only be summed into a single (x,t) space after the migration operation in each separate space, the result being in an $(x, t(0))$ space. This sequence as described is, in fact, one form of a prestack Kirchhoff migration.

Suppose we now sample a wavefield amplitude over time in a two-dimensional space with a detector at $(x_d,0)$ where x_d might be x_0 as shown in Figure 1. If we plot these samples over time as a third dimension without regard to the wavefield source (as, in fact, it may derive simultaneously from several sources), we now have a space which includes (x,t) planes in which a Kirchhoff migration would not follow any ellipses. Such planes would be essentially horizontal surfaces (as can be appreciated from Figure 2). X_d would be one of the foci of the ellipse shown in the figure. Hence, the issue of precisely what a “time” variable signifies is anything but a rhetorical matter, especially in view of some rather common industry practices.

Referring to Figure 1 once more, an independent *t* axis as described can be drawn orthogonally to the (x,z) plane (Figure 2). Each Kirchhoff ellipse corresponding to a constant time value (as well as constant distance in this case) will now appear in a separate (x,z) plane in the new (x,z,t) space.

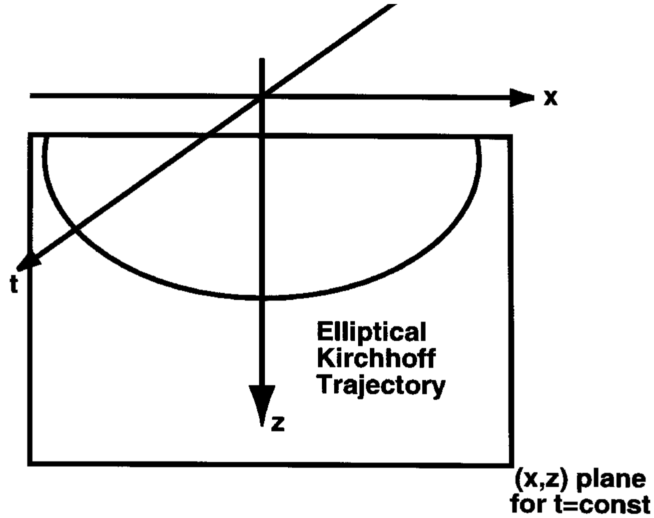

Figure 2. (x,z,t) space and (x,z) plane for constant t value.

The natural question, of course, is what would the Kirchoff migration look like in planes containing the time origin and variables *x* or *z*. Viewing the (x,t) planes for various depths *z* is very interesting since these show envelopes of the points of intersection between ellipses standing in vertical planes with parallel/horizontal (x,t) planes. Looking at the (z,t) planes is quite another matter.

The standard equation for an ellipse nicely describes all views if we substitute *t* for t_c and regard it as a variable. The views we seek to understand are achieved by then holding *z* or *x* constant in turn. Turning first as indicated to (x,t), with *z* considered fixed. Both *x* and *t* appear to the second power. Note that for the special case $x_0 = 0$ (collocated source and receiver), we have

$$t^2 = (\frac{2z}{V})^2 + (\frac{2x}{V})^2 = t_0^2 + (\frac{2x}{V})^2$$

where t_0 is defined as normal incidence time implicitly above (recall $t = t_c$). This result is the familiar hyperbolic normal moveout relation. We clearly no longer have ellipses in the (x,t) planes as now defined in this manner.

Recall that the independent time value t appears in both a and b of the standard equation for an ellipse. We intend now to hold *x* constant at x_d. Since t is the time variable, the relation which results is again rather complex. Just for comparative purposes, if *x* is set to zero in the standard equation for an ellipse, a hyperbolic curve clearly results. This is rather similar to the relationship seen in the final equation above.

Conclusion. Our main point is to understand that the Kirchhoff imaging approach is most easily understood when working with spatial coordinates. Time variables may be defined in several ways and it is important that these be clearly understood so no confusion results. The lesson here, using only two spatial variables, is readily extended to the full three-dimensional case, of course. Advanced imaging methods are best defined most generally using a true (x,y,z,t) space and not employing a time variable having any implicit or explicit dependence on other physical parameters of the system.

Corresponding author: N. Neidell, 713-975-1262, fax 713-975-0258